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LABORATORY LIMITS ON THEORIES WITH STERILE NEUTRINOS IN THE BULK

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We discuss the phenomenological consequences of theories which describe sterile neutrinos in large extra dimensions, in the so-called bulk. We briefly outline how the cumulative non-decoupling effect due to the tower of Kaluza-Klein singlet neutrinos may equivalently be described by a higher-dimensional effective theory with original order-unity Yukawa couplings. Based on this cumulative phenomenon, we obtain strong constraints on the fundamental quantum gravity scale and/or on the higher-dimensional Yukawa couplings.

It has recently been realized [1] that the experienced weakness of gravity may be ascribed to the spreading of the gravitational forces in δ (≥ 2) large extra (spatial) dimensions, namely in the so-called bulk. In such a novel scenario, the fundamental $\sim$ TeV scale of quantum gravity M_F is related to the ordinary Planck mass $M_{\rm P} = 1.2 \times 10^{19}$ GeV and the compactification scale R, through a kind of generalized Gauss' law: $M_{\rm P} \approx M_F (RM_F)^{\delta/2}$. In this framework, the Standard Model (SM) fields are confined to a 3-brane configuration, whilst gravity and most probably fields which are singlets under the SM gauge group, such as sterile neutrinos, are only allowed to propagate in the large compactified dimensions.[2,3]

For our phenomenological discussion, we shall adopt a variant [4,5] of the higher-dimensional singlet-neutrino model which was originally discussed in Refs.[2,3] For definiteness, we consider a model that minimally extends the SM-field content by one singlet Dirac neutrino, $N(x, y)$, which propagates in a $[1 + (3 + \delta)]$-dimensional Minkowski space. The y-coordinates are compactified on a circle of radius R by applying the periodic identification: $y \equiv y + 2\pi R$. For more details on this model as well as the conventions followed here, the reader is referred to Ref.[5]

The leptonic sector of the our minimal model of interest to us is given by

$$\mathcal{L}_{\rm eff} = \int\limits_0^{2\pi R} d\vec{y} \, \Big[\bar{N}\Big(i\gamma^\mu \partial_\mu + i\gamma_{\vec{y}} \partial_{\vec{y}} - m \Big) N \, + \, \delta(\vec{y}) \Big(\sum_{l=e,\mu,\tau} \frac{h_l}{M_F^{\delta/2}} L_l \tilde{\Phi} \xi + \text{h.c.} \Big) \Big], \quad (1)$$

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where $\tilde{\Phi} = i\sigma_2\Phi$ ($\langle\tilde{\Phi}\rangle = v = 174$ GeV) and L_l are the Higgs and lepton doublets, respectively. In Eq. (1), we have assumed that only one two-component spinor ξ of the higher-dimensional one N interacts with our 3 brane, while its non-interacting two-component Dirac partner is denoted by η. After integrating out the y-coordinates, we arrive at the effective Lagrangian:

$$\sum_{\vec{n}} \Bigg[\bar{\xi}_{\vec{n}}(i\bar{\sigma}^\mu\partial_\mu)\xi_{\vec{n}} + \bar{\eta}_{\vec{n}}(i\bar{\sigma}^\mu\partial_\mu)\eta_{\vec{n}} - \Big(\xi_{\vec{n}}(m + \frac{i|\vec{n}|}{R})\eta_{-\vec{n}} + \sum_{l=e,\mu,\tau} \bar{h}_l L_l \tilde{\Phi}\xi_{\vec{n}} + \text{h.c.}\Big)\Bigg] \quad (2)$$

where $|\vec{n}|^2 \equiv \sum_{i=1}^{\delta} n_i^2$, $\xi_{\vec{n}}$ and $\eta_{\vec{n}}$ are the Kaluza-Klein (KK) excitations, and $\bar{h}_l = h_l M_F/M_\mathrm{P}$ is the known 4-dimensional Yukawa-coupling suppressed by the volume factor $M_F/M_\mathrm{P} \sim 10^{-16}$.[2,3] In this minimal model, the observed light neutrinos ν_l have very small admixtures $B_{l,\vec{n}}$ with the heavy KK states $\eta_{\vec{n}}$:

$$\nu_l \approx \frac{1}{\sqrt{1+\sum_{\vec{n}}|B_{l,\vec{n}}|^2}}(\nu_{lL} + \sum_{\vec{n}} B_{l,\vec{n}}\eta_{\vec{n}})\,, \quad \text{with} \quad B_{l,\vec{n}} \approx \frac{h_l\, v M_F}{M_P\sqrt{m^2+\vec{n}^2/R^2}}\,, \quad (3)$$

where $\sqrt{m^2+\vec{n}^2/R^2}$ are approximately the masses of the physical heavy KK states.

In order to quantify the new-physics effects mediated by KK neutrinos both at the tree and one-loop levels, it proves useful to define the mixing parameters

$$(s_L^{\nu_l})^2 \equiv \sum_{\vec{n}} |B_{l,\vec{n}}|^2 \approx h_l^2 \frac{v^2}{M_F^2} \sum_{\vec{n}} \frac{M_F^4 M_P^{-2}}{(m^2 + \frac{\vec{n}^2}{R^2})} \approx \begin{cases} \dfrac{\pi h_l^2 v^2}{M_F^2} \ln\left(\dfrac{M_F^2}{m^2}+1\right), \ \delta = 2 \\[2ex] \dfrac{S_\delta}{\delta-2}\,\dfrac{h_l^2 v^2}{M_F^2}, \ \delta > 2\,, \end{cases} \quad (4)$$

where $S_\delta = 2\pi^{\delta/2}/\Gamma(\delta/2)$ is the surface area of a δ-dimensional sphere of unit radius. In deriving the last step in Eq. (4), we have approximated the sum over the KK states by an integral, with an upper ultra-violet (UV) cutoff at $M_F R$, above which string-threshold effects are expected to be more relevant.

As can be seen from Table 1, the mixings $(s_L^{\nu_l})^2$ may be constrained by a number of new-physics observables induced at the tree level. These observables measure possible non-universality effects in μ, τ and π decays. In this respect, in Table 1 we have defined $R_\pi = \Gamma(\pi\to e\nu)/\Gamma(\pi\to\mu\nu)$ and $R_{\tau\mu} = B(\tau\to e\nu\nu)/B(\tau\to\mu\nu\nu)$.

Owing to the tower of the KK singlet neutrinos which acts cumulatively in the loops, significant universality-breaking as well as flavour-violating effects are induced in electroweak processes involving γ-[8] and Z- boson [5] interactions. In particular, as has been explicitly shown recently,[5] we find that the cumulative presence of the KK states leads to an effective theory whose Yukawa interactions are mediated by order-unity Yukawa couplings of the original Lagrangian before compactification. In this case, we expect a higher-dimensional non-decoupling phenomenon analogous to the one studied earlier in renormalizable 4-dimensional theories.[6,7] For example, the effective lepton-flavour-violating vertex Zll' that occurs in $\mu \to eee$ exhibits the dependence: $\mathcal{T}(Zl'l) \propto h_l h_{l'}\,(v^2/M_F^2)\sum_{k=e,\mu,\tau}(h_k^2 v^2)/M_W^2$, i.e. its strength

Table 1. Tree-level limits on M_F/h.

Observable	$h_e = h_\mu = h_\tau = h$ Lower limit on M_F/h		$h_\mu = 0$ and $h_e = h_\tau$ Lower limit on M_F/h_τ	
	$\delta = 2$	$\delta > 2$	$\delta = 2$	$\delta > 2$
$1 - \dfrac{\Gamma(\mu \to e\nu\nu)}{\Gamma_{\rm SM}(\mu \to e\nu\nu)}$	$8.9 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{3.5\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$	$6.3 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.5\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{\Gamma(Z \to \nu\nu)}{\Gamma_{\rm SM}(Z \to \nu\nu)}$	$5.9 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.4\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$	$4.8 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{1.9\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{R_\pi}{R_\pi^{\rm SM}}$	–	–	$18.7 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{7.5\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$
$1 - \dfrac{R_{\tau\mu}}{R_{\tau\mu}^{\rm SM}}$	–	–	$5.7 \ln^{1/2} \dfrac{M_F}{m}$	$\dfrac{2.3\, S_\delta^{1/2}}{\sqrt{\delta - 2}}$

Table 2. One-loop-level limits on M_F/h^2.

Observable	$h_e = h_\mu = h_\tau = h \geq 1$ Lower limit on M_F/h^2 [TeV]		
	$\delta = 2$	$\delta = 3$	$\delta = 6$
$\text{Br}(\mu \to e\gamma)$	75	43	33
$\text{Br}(\mu \to eee)$	250	230	200
$\text{Br}(\mu\ ^{48}_{22}\text{Ti} \to e\ ^{48}_{22}\text{Ti})$	380	320	300

increases with the fourth power of the higher-dimensional Yukawa couplings. This should be contrasted with the respective photonic amplitude $\mathcal{T}(\gamma l' l) \propto h_l h_{l'}\, v^2/M_F^2$, whose strength increases only quadratically.

Based on this cumulative non-decoupling effect, we are able to derive strong limits on the M_F/h^2, for $h \geq 1$. As is displayed in Table 2, the strongest limits are obtained from $\mu \not\to eee$ and the absence of μ-to-e conversion in nuclei. Of course, the limits presented here contain some degree of uncertainty, which is inherent in all effective non-renormalizable theories with a cut-off scale, such as M_F. Nevertheless, our results are very useful, since they indicate the generic size of constraints that one has to encounter in model-building considerations with sterile neutrinos.[9]

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